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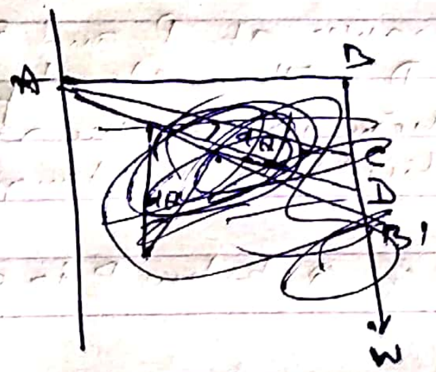
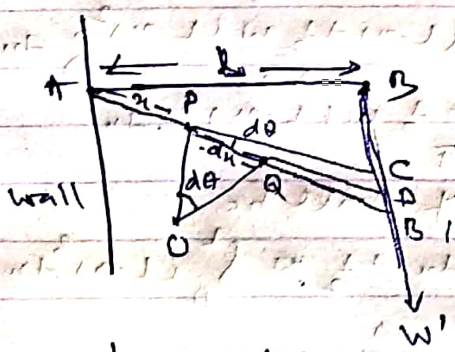
Degree - I

Expression for depression at any point when a uniform beam is fixed in one end and loaded at the other end :—

Let AB is a beam fixed at the end A ~~and~~ on the wall horizontally and other end B is loaded by a weight ~~W~~ as shown in the figure given below and P be a section at a distance x from A.



Let the point B shifts to B' due to weight W such that the weight of load be heavy to the Uniform beam. Let L be the length of AB and P be a section at a distance x from A.



Now, here in addition to the weight W, the wt of the portion (L−x) is also acting at the mid point i.e. Centre of gravity of this portion.

Thus if w be the wt per unit length, then a wt $w(L-x)$ is acting at a distance $\frac{(L-x)}{2}$ from P.

Therefore external bending moment due to the load $W' = W(L-x)$ and due to the wt $w(L-x) = w(L-x)\cdot\frac{1}{2}(L-x)$

$\therefore$ total bending moment $= W(L-x) + w(L-x)\frac{1}{2}(L-x)$

$$= W(L-x) + \frac{1}{2}w(L-x)^2 \quad \text{—(1)}$$

Since the beam is in equilibrium, this external bending moment must balanced the internal bending moment $\frac{YI}{R}$. where ~~R is the radius and~~ Y young's modulus of elasticity and R is the radius of curvature of neutral axis at P.

$$\therefore W(L-x) + \frac{1}{2}w(L-x)^2 = \frac{YI}{R} \quad \text{—(2)}$$

Now as we proceed towards the fixed end A, the moment of external bending force increases and radius of curvature also changes. But for a point Q very near to P there can be assumed to be equal. Thus if PQ = dx

then $PQ = R d\theta$

$$\text{or } \frac{1}{R} = \frac{d\theta}{PQ} = \frac{d\theta}{dx} \quad \text{--- (3)}$$

Substituting $\frac{1}{R}$ in eqn (2)

we get
$$W(L-x) + \frac{1}{2}w(L-x)^2 = YI\frac{d\theta}{dx}$$

$$\therefore d\theta = \frac{W(L-x) \frac{1}{2}w(L-x)^2}{YI} \cdot dx \quad \text{--- (4)}$$

PC and QD tangents drawn at P and Q which meant BB' in C and D. Then clearly, the angle subtended by them is $d\theta$, the radii at P and Q being perpendicular to the tangents there.

Now clearly depression of Q below P is equal to CD = dy (say)

$$\text{then } dy = (L-x)\,d\theta$$
$$= (L-x)\left\{\frac{W(L-x) + \frac{1}{2}w(L-x)^2}{YI}\right\} dx$$
$$= \frac{W(L-x)^2 + \frac{1}{2}w(L-x)^3}{YI} \cdot dx$$

Therefore total depression y will be

$$y = \int dy = \frac{W}{YI}\left\{\int_0^L (L-x)^2 dx + \frac{1}{2} \int_0^L (L-x)^3 dx\right\}$$
$$= \frac{W}{YI} \cdot \frac{L^3}{3} + \frac{1}{2}\frac{w}{YI}\frac{L^4}{4}$$
$$= \frac{WL^3}{3YI} + \frac{1}{8}\frac{wL^4}{YI}$$
$$= \frac{WL^3}{3YI} + \frac{1}{8}\frac{L^3 \cdot (wL)}{YI}$$
$$\text{or } = \frac{WL^3}{3YI} + \frac{1}{8}\frac{L^3 \cdot W'}{YI} \quad \text{Where } wL = W'$$
$$= \frac{L^3}{3YI}\left[W + \frac{3}{8} \cdot W'\right]$$

i.e. the beam now behaves as though it is loaded at the end B with a wt W plus $\frac{3}{8}$th of the wt of the beam.